SECOND TERM E-LEARNING NOTE

SUBJECT: FURTHER MATHEMATICS **CLASS: SS2**

SCHEME OF WORK

WEEK	TOPIC
1.	Differentiation: Limits of Function and First Principle, Differentiation of Polynomial
2.	Differentiation (Continued): Rules of Differentiation
3.	Differentiation of Transcendents: Derivative of Trigonometric Functions and Exponential Functions.
4.	Application of Differentiation: Rate of Change, Equation of Motion, Maximum and Minimum Points and Values of Functions.
5.	Conic Sections: Equation of Circles, General Equation of Circles, Finding Centre and Radius, Equation and Length of Tangents to a Circle.
6.	Conic Sections: The Parabola, Hyperbola and Ellipse
7.	Review of First Half Term
8.	Statistics Probability: Sample Space, Event Space, Combination of Events, Independents and Dependent Events.
9.	Permutation and Combination
10.	Dynamics: Newton's Laws of Motion
11.	Work, Energy, Power, Impulse and Momentum
12.	Revision and Examination.

REFERENCES

Further Mathematics Project 2 and 3.

WEEK ONE

TOPIC : LIMITS OF FUNCTIONS AND DIFFERENTIATION FROM THE FIRST PRINCIPLE

The followings are the properties of limits:

(i) lim k = k i e

$x^2 \to a$

The limit of a constant is the constant itself

(ii) lim [f(x) + f $_2$(x) + f$_{3\,(x)}$ + … f$_n$(x)]

= lim f$_1$(x) + lim f$_2$(x) + lim f$_3$(x) +limf$_n$(x)

$x \to a$ $x \to a$ $x \to a$ $x \to a$

i.e

The limit of the sum of a finite number of functions is equal to the sum of their respective limits

lim [f$_1$(x) – f$_2$(x)] = limf$_1$(x) – limf$_2$(x)

$x \to a$ $x \to a$ $x \to a$ $x \to a$

i.e

The limit of the difference of two functions is equal to the difference of their limits.

(iii) $\lim_{x \to a} [f_1(x)\, f_2(x)\, f_3(x) + \ldots\ f_n(x)]$

$= \lim_{x \to a} f_1(x) \lim_{x \to a} f_2(x) \lim_{x \to a} f_3(x) \lim_{x \to a} f(x)$

i.e

The limit of the product of infinite number of functions is equal to the product of their respective limits.

(iv) x a$[\frac{f(x)}{f(x)}] = \frac{\lim f_1(x)}{\lim f_2(x)}$

Provided $\lim f_2(x) \neq 0$ i.e

The limit of the quotient function is equal to the quotient of their limits provided the limit of the divisor is not equal to zero

(v) $\lim_{x \to a} k\, f(x) = k \lim_{x \to a} f(x)$

i.e

Limit of the product of a constant and a function is equal to the product of the constant and the limit of the function

Example 1

Evaluate $\lim(7 - 2x + 5x^2 - 4x^3)$

Solution

$\lim_{x \to a} \{7 - 2x + 5x^2 - 4x^3)$

$= \lim_{x \to a} 7 - 2 \lim_{x \to a} x + 5 \lim_{x \to a} x^2 - 4 \lim_{x \to a} x$

$= 7 - 0 + 0 = 7$

Example 2

$\lim_{x \to 0} \frac{x^2 + 5x + 9}{2x^2 - 3x + 15}$

Solution

$\lim_{x \to 0} \frac{x^2 + 5x + 9}{2x^2 - 3x + 15} = \frac{\lim x^2 + 5x + 9}{\lim 2x^2 - 3x + 15}$

$\frac{\lim_{x \to 0} x^2 + 5 \lim_{x \to 0} x + \lim_{x \to 0} 9}{2 \lim_{x \to 0} x^2 - 3 \lim_{x \to 0} x + \lim_{x \to 0} 15}$

$= \frac{0+0+9}{0-0+15}$

$= \frac{9}{15}$

$= \frac{3}{5}$

Example

Evaluate $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$

Solution

Lim $x^2 - 25$ = lim $\frac{(x+5)(x-5)}{x-5}$ =

x→ 5x – 5

= lim (x + 5)

x – 5

= lim x + lim 5

x – 5x – 5

= 5 + 5

= 10

Example

Evaluate lim $\frac{3x^3+2x^2+x+1}{x^3 + 2x+ 5}$

x – 5

Solution

We know that lim $\frac{1}{x}$ = 0

x – 0

lim $\frac{3x^3 + 2x^2 + x + 1}{x^3 + 2x + 5}$

x – 0

$x^3(3 + \frac{2}{x} + \frac{1}{2} + \frac{1}{3})$

lim x x = 0

x 0 x^3 (^{2} + $\frac{5}{2}$) 3

x x

lim 3 + 2 lim $\frac{1}{x}$ + lim $\frac{1}{2}$ + lim $\frac{3}{3}$

= x – 0x – 0x – 0x – 0

lim1 + 2 lim $\frac{1}{x}$ + lim $\frac{1}{2}$ + 5 lim

x – 0x – 0x – 0x – 0

= $\frac{3+ 0 + 0 + 0}{1 + 0 + 0}$

= $\frac{3}{1}$

= 3

EVALUATION

Evaluate lim -> 4 x^3 +4x 6

Evaluastelim x -> -2 x+6/ 2x +4

Differentiation From first Principle

The technique adopted in unit 11.3 in finding the derivative of a function from the consideration of the limiting value is called **differentiation from first principle.**

Example

Find the derivative of f(x) = x^2 from first principle.

Solution

f (x) = x^2

f(x + Δx) = (x + Δx)^2

= $x^2 + 2x\Delta x + (\Delta x)^2$

f(x + Δx) – f (x) = $(x + \Delta x)^2 - x^2$

= $x^2 + 2x\Delta x + (\Delta x)^2 - x^2$

$= 2x\Delta x + (\Delta x)^2$

$\frac{f(x+\Delta x)-f(x)}{\Delta x} = 2x + \Delta x$

$\lim_{\Delta \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = 2x$

$\therefore f^1(x) = 2x$

Example

Find the derivative of $y = x^3$ from first principle

Solution