FIRST TERM E –LEARNING NOTE

SUBJECT: FURTHER MATHEMATICS **CLASS: SS2**

FIRST TERM SCHEME OF WORK

WEEK	TOPIC
1	Finding quadratic equation with given sum and product of roots, conditions for equal roots, real roots and no root
2	Tangents and Normals to Curves
3	Polynomials ;definition, basic operations + , x , - , ;--
4	Polynomials (Continued) factorization
5	Cubic Equation , roots of cubic equations
6	Review and Test
7	Logical Reasoning ; fundamental issues and definitions and theorem proving
8	Trigonometric Function , six trig functions of angles of any magnitude (sine, cosine,tangent,secant, cosecant, cotangent)
9	Relationship between graph of trigonometric ratios such as sin x and sin 2x, graphs of y= a sin (bx) + c , y = a cos (bx) + c , y = a tan (bx) + c
10	Graphs of inverse by ratio and equation of simpletrgonometric identities
11	Revision

REFERENCES

- Further Mathematics Project 1 by TuttuhAdegun
- Further Mathematics Project 2 by TuttuhAdegun
- Additional Mathematics by Godman

WEEK 1
TOPIC: SOLUTION TO QUADRATIC EQUATION
FINDING QUADRATIC EQUATION GIVEN SUM AND PRODUCT OF ROOTS CONDITION FOR EQUAL ROOTS, REAL ROOTS AND NO ROOT

We recall that if $ax^2 + bx + c = 0$, where a, a and c are constants such that $a \neq 0$, then,
$x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ or $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$

Suppose we represent these distinct roots by α and β; thus:
$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$
and
$\beta\frac{-b-\sqrt{b^2-4ac}}{2a}$

We may also put $D = b^2 - 4ac$, so that
$\alpha = \frac{-b+\sqrt{D}}{2a}$
$\beta = \frac{-b-\sqrt{D}}{2a}$

Sum of roots
$\alpha + \beta = \frac{(-b+\sqrt{D})}{2a} + \frac{(-b-\sqrt{D})}{2a}$
$= \frac{-2}{2b}$

$= \frac{-b}{a}$
Products of roots
$\alpha\beta = \frac{(-b+\sqrt{D})\,(-b-\sqrt{D})}{2a\ X\ 2a}$
$\because \alpha\beta = \frac{b^2 - D}{4a^2}$
$= \frac{b^2 - (b^2 - 4ac)}{4a^2}$
$= \frac{4ac}{4a^2}$
$= \frac{c}{a}$
Hence, if $ax^2 + bx + c = 0$, where a, b and c are constants and$\alpha \neq 0$ then $\alpha + \beta = \frac{-b}{a}$,
$\alpha\beta = \frac{c}{a}$, *we recall from* 5.3 *that by the method of factorization if*
$x^2 + x - 42 = 0$
then $(x - 6)(x - 7) = 0$

Hence the roots of the equation are 6 and -7. In general, if a quadratic equation factorizes into
$(x - \alpha)(x - \beta) = 0$
then α and β must be the roots of that equation.

The general quadratic equation $ax^2 + bx + c = 0$ can also be written as:
$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$ ...(1)

If the roots of the equation are α and β then the above equation can be written as:
$(x - \alpha)(x - \beta) = 0$
$x^2 - (\alpha - \beta)x + \alpha\beta = 0$ ---(2
By comparing coefficients in equations (1) and (2)
$-(\alpha + \beta) = \frac{b}{a}$

: $\alpha + \beta = \frac{-b}{a}$

and$\alpha\beta = \frac{c}{d}$

The above consideration gives rise to two problems:

(a) Given a quadratic equation, we can find the sum and product of the roots.

(b) Given the roots, we can formulate the corresponding quadratic equation.

The quadratic equation whose roots are α and β is

$x^2 - (\alpha + \beta) x + \alpha \beta = 0$

Find the sum and product of the roots of each of the following quadratic equations:

(a) $2x^2 + 3x - 1 = 0$

(b) $3x^2 - 5x - 2 = 0$

(c) $x^2 - 4x - 3 = 0$

(d) $\frac{1}{2} x^2 - 3x - 1 = 0$

Solution

(a) $2x^2 + 3x - 1 = 0$

a = 2; b = 3; c = -1

Let α and β be the roots of the equation, then

$\alpha + \beta = \frac{-b}{a} = \frac{-3}{2}$

$\alpha \beta = \frac{c}{a} = \frac{-1}{2}$

(b) $3x^2 - 5x - 2 = 0$

a = 3; b = -5; c = -2

Let α and β be the root of the equation, then

$\alpha + \beta = \frac{-b}{a} = \frac{5}{3}$

$\alpha \beta = \frac{c}{a} = \frac{-2}{3}$

(c) $x^2 - 4x - 3 = 0$

a = 1; b = 4; c = -3

Let α and β be the root of the equation, then

$\alpha + \beta = \frac{-b}{a} = \frac{4}{1}$

$\alpha \beta = \frac{c}{a} = -3$

(d) $\frac{1}{2} x^2 - 3x - 1 = 0$

a = ½, b = -3, c = -1

Let α and β be the root of the equation, then

$\alpha + \beta = \frac{-b}{a} = \frac{(3)}{\frac{1}{2}} = 6$

$\alpha \beta = \frac{c}{a} = \frac{-1}{\frac{1}{2}}$ = -2

Find the quadratic equation whose roots are:

(a) 3 and -2 (b) ½ and 5

(c) -1 and 8 (d)¾ and ½

Solution

The quadratic equation whose roots are…