SUBJECT: FURTHER MATHEMATICS

CLASS: SS1

SCHEME OF WORK

WEEK	TOPIC
1	Arithmetic Progression (AP)
2	Geometric Progression (GP)
3	Linear inequalities in one variable
4	Inequalities in two variables (Graph of inequalities)
5	Introduction to the concept of functions.
6	Review of half term work.
7	Functions (one – to – one, onto, composite and inverse functions)
8	Trigonometric ratio: Graph of Sine, Cosine and tangent of angles, deviation of trigonometric ratio of special angles (30^0, 45^0 and 60^0). Application of trigonometric ratios.
9	Logical reasoning: Simple True and False statement, Negation, Converse and Contra positive of statement,
10	Logical reasoning continues: Compound statement, connectives and their symbols, conditional statements and symbols.
11	Revision of Second Term's lesson
12	Examination

REFERECES
➢ FutherMaths Project 1 and 2 by TuttuhAdegun (main text).
➢ Additional Mathematics by Godman
➢ Further Mathematics by E. Egbe et al.

WEEK ONE   **DATE……………**
TOPIC: SEQUENCE & SERIES
CONTENT
- Sequence and series
- Arithmetic Progression (AP)
- Arithmetic Mean
- Sum of terms in an AP

Sequence & Series
A sequence is a pattern of numbers arranged in a particular order. Each of the number in the sequence is called a term. The terms are related to one another according to a well defined rule.

Consider the sequence 1, 4, 7, 10, 13 …., 1 is the first term,(T_1) 4 is the second term(T_2), 7 is the third term (T_3).
The sum of the terms in a sequence is regarded as series. The series of the above sequence is
1 + 4 + 7 + 10 + 13 = 35

The nth term of a Sequence
The nth term of a sequence whose rule is stated may be represented by T_n so that T_1, T_2, T_3 etc represent the first term, second term, third term … etc respectively.
Consider the sequence 5, 9, 13, 17, 21 ……..
$T_1 = 5 + 4(0)$
$T_2 = 5 + 4(1)$
$T_3 = 5 + 4 (2)$

$T_4 = 5 + 4 (3)$
$T_n = 5 + 4 (n - 1)$
$T_n = 5 + 4n - 4 \quad = 4n + 1$
when $n = 30$
$T_{30} = 4(30) + 1$
$T_{30} = 121$

Find the nth term of these sequences:
(i)   3, 5, 7, 9 …… 2n + 1
(ii) 0, 1, 4, 9 ……… $(n - 1)^2$
(iii) 1/3, 3/4, 1, 7/6 ………………$\dfrac{2n - 1}{n + 2}$

Examples
Write down the first four terms of the sequence whose general term is given by:
(i) $T_n = \dfrac{n+1}{3n + 2}$ (ii) $T_n = 5 \times (\tfrac{1}{2})^{n-2}$

$$T_4 \quad = \quad \frac{4 + 1}{} \quad = \frac{5}{14}$$

Solution
i. $T_n \quad = \quad \dfrac{n+1}{3n + 2}$

$T_1 \quad = \quad \dfrac{1 + 1}{3(1) + 2} = \frac{2}{5}$

$T_2 \quad = \quad \dfrac{2 + 1}{3(2) + 2} = \frac{3}{8}$

$T_3 \quad = \quad \dfrac{3 + 1}{3(3) + 2} = \frac{4}{11}$

$\qquad 3(4) + 2$

The sequence is $\frac{2}{5}, \frac{3}{8}, \frac{4}{11}, \frac{5}{14}$ ……..

(ii) $T_n = 5 \times (1/2)^{n-2}$
$T_1 = 5 \times (1/2)^{1-2} = 5(\tfrac{1}{2})^{-1} = 5(2^{-1})^{-1} = 5 \times 2 = 10$
$T_2 = 5 \times (1/2)^{2-2} = 5(\tfrac{1}{2})^{0} = 5 \times 1 = 5$
$T_3 = 5 \times (1/2)^{3-2} = 5 \times (\tfrac{1}{2}) = \frac{5}{2}$
$T_4 = 5 \times (1/2)^{4-2} = 5(\tfrac{1}{2})^{2} = \frac{5}{4}$
The sequence is $10, 5, \frac{5}{2}, \frac{5}{4}$ ………

Evaluation
Find the first term of the sequence whose general term is given by
(i) $50 - (½)^n$ (ii) $2 + \frac{3}{2}^{(n+1)}$

Arithmetic Progression (A.P) or Linear Sequence
An arithmetic progression (A.P) is generated by adding or subtracting a constant number to a preceding term to get a term. This constant number is called the common difference designated by the letter d. The first term is designated by a.

Ex:	A.P	d (common difference)	a (first term)
	6½, 5, 3½, 2 ….	-1½	6½
	-2, $-\frac{3}{4}$, ½, 1 ¾	1¼	-2

T_1	T_2	T_3	T_4	T_5
a	a + d	a + 2d	a + 3d	a + 4d

So for any A.P, the nth term $(T_n = U_n)$ is given by
$\qquad\qquad T_n = U_n = a + (n - 1)\, d.$ $T_n = U_n =$ nth term
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ a = first term
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ d = common difference
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ n = no of terms

Examples
1. What is the 10th term of the sequence 10, 6, 2, -4 …..
2. Find the term of the A.P 3½, 7, 10½ ….. Which is 77.
3. The fist term of an A.P is 3 and the 8th term is 31. Find the common difference.

Solution

(1.)   The A.P = 10, 6, 2, -4
        $a = 10$, $d = 6 - 10 = -4$, $n = 10$
        $T_n = a + (n - 1) d$
        $T_{10} = 10 + (10 - 1) (-4)$
        $T_{10} = 10 + 9(-4) = 10 - 36$
        $T_{10} = -26$.

(2.)   A.P = $3\frac{1}{2}$, 7, $10\frac{1}{2}$ 77
        $a = 3\frac{1}{2}$, $d = 7 - 3\frac{1}{2} = 3\frac{1}{2}$, $n = ?$ $T_n = 77$
        $T_n = a + (n-1)d$
        $77 = 3\frac{1}{2} + (n-1)3\frac{1}{2}$
    $77 = 3\frac{1}{2} + 3\frac{1}{2}n - 3\frac{1}{2}$
        $77 = 3\frac{1}{2} n$
        $n = {}^{77}/_{3\frac{1}{2}} = {}^{77}/{}^{7}/_{2}$
        $n = {}^{77} \times {}^{2}/_{7} = 22$

(3) $a = 3$, $T_8 = 31$, $d = ?$ $n = 8$
$T_n = a + (n-1) d$
    $31 = 3 + (8-1) d$
        $31 - 3 = 7d$
          $d = {}^{28}/_{7} = 4$

Evaluation
(i)   Find the 15$^{\text{th}}$ term of the A.P 5, 2, -1, -4 …………
(ii)   Find the term of the A.P 1, 6, 11, 16…. which is 66.

Arithmetic Mean
If a, b, c are three consecutive terms of an A.P, then the…