THIRD TERM E-LEARNING NOTE

SUBJECT: FURTHER MATHEMATICS **CLASS: SS 2**

SCHEME OF WORK

WEEK	TOPIC
1.	Revision of Second Term Examination Questions.
2.	Projectile: Trajectory of Projectile, Greatest Height Reached, Time of Flight, Range, and Projectile along Inclined Plane.
3.	Binomial Expansion: Pascal Triangle Binomial Theorem of Negative, Positive and Fractional Power
4.	Mechanics: Vectors in Two and Three Dimension. Scalar Product of Vectors inThree Dimension.
5	Vector or Cross Product on Three Dimension.Application of Cross Product Cross Product of Two Vectors.
6	Review of the Half Term Work.
7.	Integration: Indefinite Integrals Concept, Different Methods of Integration.e.g (Algebraic and Trigonometric Substitution by Parts and Partial Fractions.
8.	Integration Continued: Definite Integral, Area Under Curve.
9.	Integration Continued: Application of Integration to Kinematics Volumes of Solids at Revolution and Trapezium Rule.
10.	Correlation and Regression: Concept, Scatter Diagram, Regression Line, Coefficient of Regression, Rank Correlation and Product Moment Correlation Coefficient.
11.	Revision.

REFERENCES
Further Mathematics Project 2 and 3.

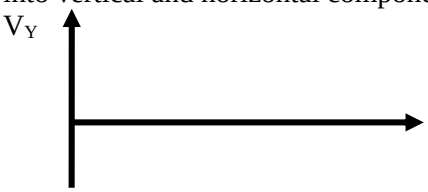
WEEK ONE
REVISION OF SECOND TERM EXAMINATION QUESTIONS.

WEEK TWO

TOPIC:PROJECTILES: MOTION UNDER GRAVITY IN TWO DIMENSION,DERIVATION AND APPLICATION OF EQUATIONS INVOLVING GREATEST HEIGHT, TIME OF FLIGHT AND RANGE

Motion Under Gravity in Two Dimensions:

If a particle is projected with an initial velocity u at angle $\boldsymbol{\theta}$ to the horizontal, the prativle will be resolved into vertical and horizontal components of the velocity.



Horizontal components: Vx = ucos$\boldsymbol{\theta}$

Horizontal distance: Sx = utcos$\boldsymbol{\theta}$

Vertical components: Vy = usin$\boldsymbol{\theta} - gt$

Vertical distance: sy = utsin$\boldsymbol{\theta}$ - ½ gt^2

Magnitude of the velocity, v = $\sqrt{v_x^2 + v_y^2}$

The acceleration due to gravity acts against the motion of the body, hence it is negative.

Example:

1. A particle is projected with an initial velocity of 46m/s at an angle of 55^0 to the horizontal. After 3 seconds, find: (i) the vertical component of the velocity (ii) horizontal component (iii) vertical distance traveled. (iv) Magnitude of the velocity.

   SOLUTION:

   $\boldsymbol{\theta}$ = 55^0 u=46m/s

   (i) Vy = usin$\boldsymbol{\theta}$- gt

   = 46 sin 55 – 10 x 3

   = 37.68 – 30

   = 7.68/s

   (ii) **Vx = ucosθ**

   = 46 cos 55

   =26.38m/s

   (iii) **sy = utsinθ - $\frac{1}{2}$ (10x9)**

   = 138sin55 – 5x9

   = 113.04 – 45

   = 68.04m

   (iv) =$\sqrt{}$ $vx^2 + vy^2$

   $7.68^2 + 26.38^2$ = $\sqrt{}$58.98 + 695.9

   = 27.48m/s

   EVALUATION: A particles is fired with an initial speed of 40m/s at an angle of 30^0 to the horizontal. Determine the vertical and horizontal components of the velocity after 2.5 seconds.

   GREATEST HEIGHT REACHED: when a projected particle reaches its greatest height, the vertical components become zero. Therefore;

   Recall**Vy, = usinθ – gt**

   Squaring both sides, $(Vy)^2 = (usin\theta n - gt)^2$

   $Vy^2 = u^2sin^2\theta - 2gs_y$

   Since: vy =0, hence, $0 = u^2sin^2\theta - 2gs_y$

   $S_y = u^2sin^2\boldsymbol{\theta}$

2g

Therefore the greatest height reaches is represented by $H=\frac{u^2\sin^2\theta}{2g}$

Time taken to reach the greatest height: The time taken to reach the maximum height I at the point when the vertical component is zero. Hence,

,Vy = usinθ–gt

0 = usinθ–gt

$T=\frac{u\sin\theta}{g}$

Example:

1. A particle is projected with velocity 56m/s at an angle of 60^0 from a point O on a horizontal plane. The particle moves freely under gravity and hits the plain again A. Calculate, correct to 3 significant figures: (a) the greatest height above OA attainedby the particle (b) the time taken by the particle to reach A from O.

Solution:

U = 56m/s $\theta = 60^0$

(a) Greatest height reached, $\mathbf{h = \frac{U^2\sin^2 O}{2g}}$

$$h = \frac{56^2 \times (\sin 60)^2}{2 \times 9.8}$$

$$h = \frac{2352}{19.6} \qquad h = 120m.$$

(a) Time taken to reach A from O; $\mathbf{t = \frac{u\sin\theta}{g}}$

$$t = \frac{56 \sin 60}{9.8}$$

$$T = 4.9secs.$$

Evaluation: A project is fired with a velocity of 45m/s and at angle of elevation of 81^0 to the horizontal. Find the time taken by the particle to reach its destination. (Take g = $10m/s^2$)

Time of flight: This is the time taken by a particle which is projected to return to its original point of projection. At this point the vertical distance becomes zero. Hemce,

$T = \frac{2u\sin\theta}{g}$

Range: This is the horizontal distance covered when the particle returns to its original point of projection. The range is equal to the product of the horizontal component and the time of flight.

Hence,

R =ucosθ$\times\frac{2u\sin\theta}{g}$

$R = \frac{u^2 \times 2\sin\theta\cos\theta}{g}$ (but; $2\sin\theta\cos\theta = \sin 2\theta$)

$R = \frac{u^2 \times 2\sin\theta}{g}$

Maximum range: A particle will cover a maximum range if it is projected at angle 45^0 to the horizontal. That is; $\theta = 45^0$. Thus $\sin 2\theta = 1$

Hence, $R_{max} = \frac{U^2}{g}$

Example: The vertical and horizontal components of the initial velocity of a projectile are 36m/s and 64m/s. find (i) initial velocity of the projectile (ii) the inclination to the horizontal at which the projectile was fired. (iii) the greatest height reached; (iv) the time of flight; (v) the horizontal range of the projectile.

Solution: